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RESEARCH ARTICLE

Hysteresis effect on sediment rating curves

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Abstract

For steady flows, a unique relationship between the sediment discharge and water discharge exists, but for unsteady flows this relationship follows a hysteresis loop. The conventional regression approach does not account for this hysteresis effect. In this study, experiments were carried out on a tilting flume under unsteady flow conditions. Two uniform sizes of sand viz., 0.3 mm and 0.35 mm were used in the study. The dimensional analysis of the variables affecting the flow was carried out and a functional relationship established by regression analysis. Two equations for rising and falling stages of hydrograph were obtained, and their validity tested on independently observed experimental data. The predicted and observed sediment discharge was found to be in good agreement.

Keywords: Sediment discharge, hysteresis loop, tilting flume, unsteady flow conditions, regression analysis.

Introduction

Sediment rating curves are widely used to estimate the sediment load being transported by a river. A sediment rating curve is a relation between the sediment and river discharges. Such a relationship is usually established by regression analysis, and the curves are generally expressed in the form of a power-law type equation. A number of equations are available to relate the amount of sediment transported by a river with flow conditions such as discharge, velocity, and shear stress. However, none of these equations have received universal acceptance. Usually, either the sediment discharge or the sediment concentration is related to the water discharge. Karim and Kennedy (1990) attempted to establish relations among the velocity, sediment discharge, bed-form geometry, and friction factor of alluvial rivers. Lopes and Ffolliott (1993) pointed out that an additional complexity is introduced to the sediment concentration and stream flow relationship due to a hysteresis effect. The sediment concentrations for a given level of stream flow discharge in the rising stage of a stream flow hydrograph are greater than those corresponding to the falling stage. The conventional regression approach is not able to account for this hysteresis effect.

Sediment rating curves

The statistical relationship between suspended sediment concentration or load and stream discharge is called the rating curve (Syvitski *et al.,* 2000) and commonly takes the power law form:

$$C_s = aQ^b \qquad (1)$$

Where, C_s = suspended sediment concentration; Q = discharge; and a and b are sediment rating coefficient and exponent, respectively.

The suspended load Q_s of a river is similarly related to the discharge by the same rating coefficients,

$$Q_s = aQ^{b+1} \qquad (2)$$

As the discharge is not measured very frequently, in many cases, the estimation of sediment being transported is a two-step procedure. The measured stage data are used to estimate discharge which, in turn, is used to estimate the sediment concentration. A sediment rating curve is similar to a discharge rating curve, except that the relationship is established between water discharge and sediment concentration or sediment discharge.

Functional relationship

The variables affecting the sediment transport phenomenon under unsteady flow conditions, were identified as q_s the sediment discharge per unit width, U the mean velocity of flow, h the depth of flow, ρ the mass density of fluid (water), $\Delta\rho_s = (\rho_s - \rho)$ in which ρ_s is the mass density of sediment particles, g the acceleration due to gravity, μ the dynamic viscosity of the fluid (water), d the diameter of sediment particles, $\Delta h/\Delta t$ the rate of change of flow depth, and S the energy slope.

The dimensional analysis was carried out and the functional relationship was expressed in the form

$$\psi = \frac{q_s}{\rho_s g^{3/2} d^{3/2}\sqrt{\dfrac{\Delta\rho_s}{\rho}}} = f\left(\frac{U}{\Delta h/\Delta t}, \frac{S}{\Delta\rho_s/\rho}, \frac{h}{d}\right) \qquad (3)$$

Therefore, in this study, the relationship was sought among the four dimensionless terms of Eq. (3).

Materials and methods

Experimental details: The experiments were carried out in the Fluid Mechanics laboratory of the N.I.T, Srinagar, J and K, India. A 24 m long, 1 m wide and 60 cm deep, glass walled, rectangular tilting flume with sediment feeder at the upstream end, was used for the study. An overhead tank supplied water to the flume through a 200 mm diameter pipe, and after flowing through the flume the water was led to an underground sump via an escape channel. Water discharge was measured by a sharp crested weir installed in the escape channel. The formula proposed by Ranga Raju and Asawa (1977) was used for calculating the water discharge over the sharp crested rectangular weir.

Two uniform sizes viz., 0.3 mm and 0.35 mm, of sand were used in the experiments. A middle portion of the flume length was selected as the working section in which near uniform flow was achieved, under steady state conditions, for almost the entire range of water discharges used in the experiments. Three water level sensors were installed in the working section which were placed 3 m apart. Another water level sensor was installed in the escape channel for the measurement of head upstream of the crest of the sharp-crested rectangular weir which was used for the measurement of water discharge. The water level data of all the four sensors were recorded in a data logger and latter retrieved on a computer using the relevant software. A sediment concentration probe was installed at the centre of cross section, in the downstream portion of the working section. This probe was connected to a meter which gave the digital display of the sediment concentration in g/L. Some experiments were conducted for steady flow of water and the values of suspended sediment concentrations recorded, for the entire range of water discharges used in the experiments. During the unsteady flow experiments, sediments of the same size, as laid on the bed of the flume, were fed near the upstream end of the flume, from the sediment feeder. The rate of sediment feed was kept equal to the sediment transport rate for an equivalent steady flow. The rate of sediment feed was varied manually by varying the size of the slit opening of the sediment feeder. Fourteen hydrographs (*H*-1 to *H*-14) were then passed through the flume, and the measurements of water discharge, water depths and sediment concentration made. The inflow into the flume was manually regulated with the help of valves located in the supply line coming from the constant head tank. The experiments were repeated for different bed slopes which were obtained by tilting the flume. The bed slope of the flume ranged from 0.002 to 0.009. The whole procedure was repeated for the second size of the bed material. During all the experimental runs the temperature of water was measured, with the help of a digital thermometer, for the purpose of determining the relevant physical properties of water.

For validation of results, another set of fourteen hydrographs *H*-15 to *H*-28, were passed through the flume. The hydrographs *H*-15 to *H*-28 were passed through the flume, using arbitrary slopes and for some hydrographs 0.3 mm material size was used, whereas, for the remaining hydrographs, 0.35 mm material size was used.

Results

Regression analysis

Three or more variables can be related by using the technique of multiple regression analysis (Chow, 1964), the general equation of which may be expressed in the form:

$$x_1 = f(x_2,x_3,\ldots\ldots.,x_m) \qquad (4)$$

Where, $x_1,x_2,x_3,\ldots\ldots.,x_m$ are m variables, x_1 being the dependent variable. If Eq. (4) is linear, the regression is termed as multiple linear regression and expressed in the following form:

$$x_1 = B_1+B_2x_2+\ldots\ldots\ldots\ldots\ldots.+B_mx_m \qquad (5)$$

Where, $B_1,B_2,\ldots\ldots\ldots..,B_m$ are unknown parameters. The degree of correlation of a dependent variable with other independent variables is measured by the coefficient of multiple correlation which is given by

$$R_1 = \frac{s_e}{s} \qquad (6)$$

Where s_e is standard deviation of values of dependent parameter x_1 estimated by using the regression equation, and s is the standard deviation of the observed values of x_1. Another coefficient representing the part of the variance in the dependent variable which has been mathematically accounted for is called as the coefficient of determination. The coefficient of determination D_1 is defined as the square of the multiple correlation coefficient, and is expressed as R_1^2.

The power law type regression equation (Eq. 7) used in the study, was transformed to linear form of Eq. (5) by using logarithmic transformation.

$$\frac{q_s}{\rho_s g^{3/2} d^{3/2} \sqrt{\dfrac{\Delta\rho_s}{\rho}}} = b_1 \times \left[\frac{U}{\Delta h/\Delta t}\right]^{b_2} \times \left[\frac{h}{d}\right]^{b_3} \times \left[\frac{S}{\Delta\rho_s/\rho}\right]^{b_4} \qquad (7)$$

The experimentally observed data were used to determine the values of the unknown parameters $b_1,b_2,,b_3$, and b_4, for rising as well as falling stages. Consequently, two equations, for rising and falling stages, were developed and are given below as Eq. (8) and Eq. (9)

$$\frac{q_s}{\rho_s g^{3/2} d^{3/2}\sqrt{\frac{\Delta\rho_s}{\rho}}} = 5.316\times10^{-6}\times\left[\frac{U}{\Delta h/\Delta t}\right]^{0.192}\times\left[\frac{h}{d}\right]^{2.465}\times\left[\frac{S}{\Delta\rho_s/\rho}\right]^{0.343} \qquad (8)$$

for rising stage

$$\frac{q_s}{\rho_s g^{3/2} d^{3/2}\sqrt{\frac{\Delta\rho_s}{\rho}}} = 8.51\times10^{-12}\left[\frac{U}{\Delta h/\Delta t}\right]^{0.416}\times\left[\frac{h}{d}\right]^{4.13}\times\left[\frac{S}{\Delta\rho_s/\rho}\right]^{0.227} \qquad (9)$$

for falling stage.

For rising stage, the coefficient of multiple correlation R_1 was obtained as 0.933, and the coefficient of multiple determination D_1 was obtained as 0.8702. For falling stage, multiple correlation coefficient R_1 of 0.9872 and Coefficient of multiple determination D_1 of 0.9745**,** was obtained.

Fig. 1a-c. Comparison of computed and observed values of q_s for a typical set of hydrographs (Ahanger *et al.*, 2008).

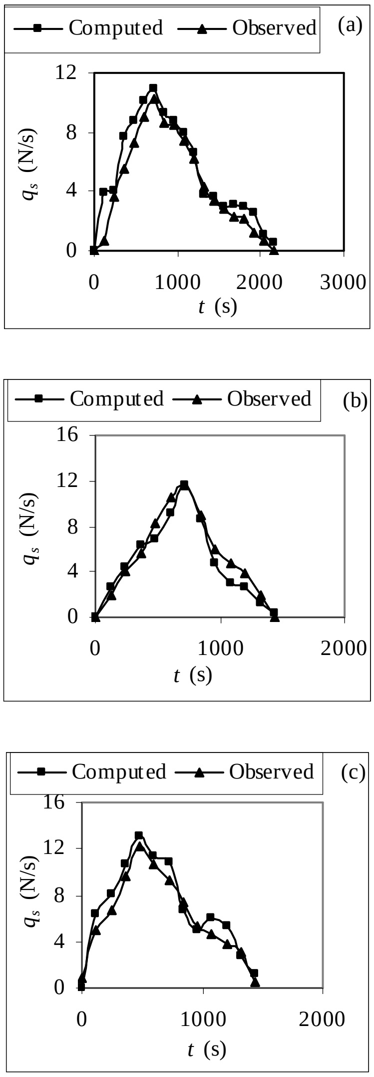
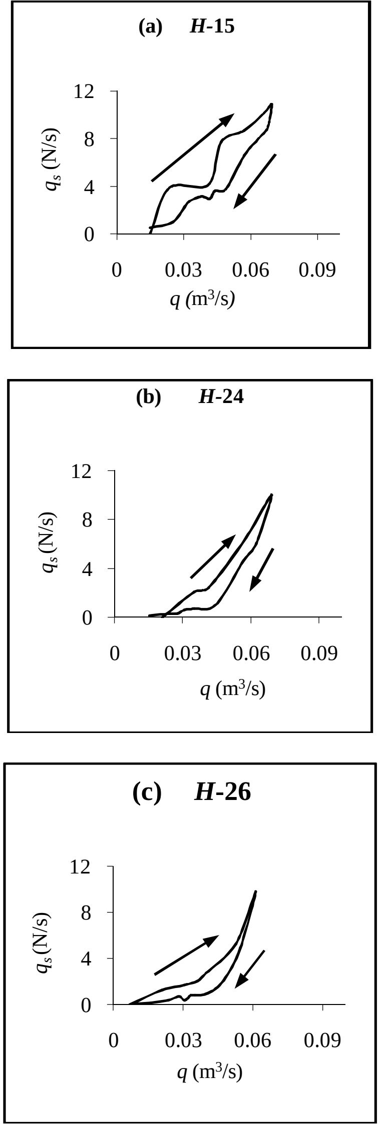


Fig. 2a-c. Computed hydrographs showing hysteresis in suspended sediment transport (Ahanger *et al.*, 2008).



Discussion

An independently observed set of hydrographs (*H*-15 to *H*-28) were used for validation of Eqs. (8) and (9). The values for the L.H.S. of the Eqs. (8) and (9) were computed after substituting the values of relevant parameters in the R.H.S. of these equations, and consequently the values of q_s were obtained. These computed values of q_s were compared with the observed q_s values and the two were found to be in good agreement. Figure 1a-c shows the comparison between observed and computed values of q_s for a typical set of hydrographs. From Figure 1a-c, it is evident the peak sediment discharge and the time to peak were modeled with reasonably good accuracy. The phenomenon of hysteresis in suspended load transport was also modeled by the Eqs. (8) and (9). Figure 2a-c shows some of the hydrographs computed by the regression Eqs. (8) and (9) wherein the hysteresis effect is prominent.

Conclusion

From the experimental study the following conclusions can be drawn:

1. Hysteresis effect on the rate of sediment transport for unsteady flow condition was established.
2. The rate of sediment transport during the rising stage of flow hydrograph is modeled by Eq. (8) and is more than the transport rate for the equivalent steady flow conditions.
3. The sediment transport rate during the falling stage of flow hydrograph is modeled by Eq. (9) and is less than the rate of sediment transport for the equivalent steady flow conditions.

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